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Elastic Response of Generic Space Vehicle Having Degraded Interstage Joints

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Introduction

MODERN high-performance launch vehicles experience significant transient elastic response of the lightly damped structure, due to external/control disturbances. Therefore, there is a need to characterize the elastic response with a greater degree of certainty. Recently, the author has attempted to improve the structural dynamic predictions in the case of generic space vehicles by including the effects of tip steady drag and trajectory acceleration.[1–3] These studies have established that there are significant differences in the structural dynamic characteristics when such effects are included in the analysis. Other important parameters that affect the structural dynamic response in space vehicles are the flexibility and location of interstage joints, which have the potential to influence the overall structural response. The present study investigates the problem of transient elastic response of generic space vehicles having degraded interstage joints and subjected to impulse excitation.

Structural Dynamic Formulation and Solution

Figures 1a and 1b show the idealized model of a normalized space vehicle structure in the form of a number of normalized constant bending stiffness EI and mass per unit length ρA steps along the length. In general, such a normalized space vehicle structural geometry can be considered to be grossly representative of geometric distributions that are normally encountered in practical space vehicles.[4] Therefore, a normalized vehicle is taken as a generic space vehicle for investigating the sensitivity of structural response to degraded interstage joints.

The elastic vibration of a space vehicle can be formulated using a stepped slender beam approach[2,5] by a set of nondimensional constant coefficient Euler–Bernoulli ordinary differential equations:

$$\frac{d^4 w_i}{dx_i^4} + \gamma_i^4 w_i = 0 \tag{1}$$

Here subscript i is the ith constant property segment of the structure; γ_i is the dimensionless frequency parameter for the ith segment, i.e., $\gamma_i^4 = [(\rho A)_i \omega^2 L_0^4/(EI)_i]$; $\bar{x}_i (= x_i/L_0)$ is the dimensionless length coordinate of each of the segments, taking values from 0 to $\bar{L}_i (= L_i/L_0)$; and w_i is the transverse elastic deformation function in each of these segments. It is usually convenient to define a single frequency parameter λ for the complete space vehicle as

$$\lambda^4 = \left[\frac{(\rho A)_0 \omega^2 L_0^4}{(EI)_0}\right] \tag{2}$$

where $(\rho A)_0$ and $(EI)_0$ are unity in the present case. The general solution can be written as[2]

$$w_i = A_i \cosh \gamma_i x_i + B_i \sinh \gamma_i x_i + C_i \cos \gamma_i x_i + D_i \sin \gamma_i x_i \tag{3}$$

where A_i, B_i, C_i, and D_i are arbitrary constants defining the deformation in the ith segment. There are a total of $4N$ such unknowns for N geometric segments, and the solution for natural frequencies and mode shapes can be obtained by enforcing the four boundary conditions on shear force and bending moments at the two free ends and $4(N-1)$ joint continuity conditions at $(N-1)$ segment junctions. The four free–free boundary conditions are

$$w_1''(0) = w_1'''(0) = w_N''(\bar{L}_N) = w_N'''(\bar{L}_N) = 0 \tag{4}$$

One of the ways to introduce the joint flexibility is through a pure rotational spring. This is reasonable because joint spans are small, leading to appreciable elastic rotation with very little elastic displacement.[6] Such an approach makes the displacement, bending moment, and shear force continuous and the slope discontinuous across a joint. In such a formulation, the equilibrium is exactly satisfied in the beam theory context, whereas compatibility is only approximately satisfied inasmuch as second and higher derivatives of displacement are discontinuous at the junction.[6] In the present study, it is considered adequate to model the interstage joint as a

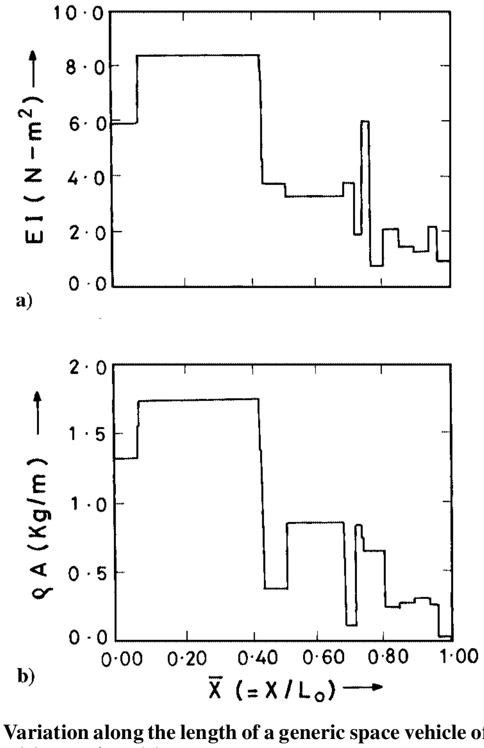

Fig. 1 Variation along the length of a generic space vehicle of normalized a) $EI(x)$ and b) $\rho A(x)$.

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point at the junction of the two beam segments. Thus, the joint continuity conditions are obtained as

$$w_i(\bar{L}_i) = w_{i+1}(0) \tag{5}$$

$$w_i'(\bar{L}_i) = w_{i+1}'(0) - \bar{F}_j \cdot w_{i+1}''(0) \tag{6}$$

$$w_i''(\bar{L}_i) = w_{i+1}''(0) \tag{7}$$

$$w_i'''(\bar{L}_i) = w_{i+1}'''(0) \tag{8}$$

The simultaneous solution of Eqs. (4–8) results in a $4N \times 4N$ characteristics determinant involving λ as the unknown parameter. Zeros of this determinant give various values of the natural frequency parameter λ_i corresponding to the ith vibration mode. The corresponding displacement function $w_i(\bar{x}_i)$ is obtained in each beam segment by solving $(N-1)$ simultaneous equations in terms of any one unknown, e.g., A_1. Finally, the function $w_i(\bar{x})$ is assembled from $w_i(\bar{x}_i)$ from each segment and is normalized by making $w_i(\bar{x}_i = 1.0)$ unity.

Transient Impulse Response Formulation and Solution

The elastic response of a lightly damped structure in its ith mode can be obtained in the time domain by enforcing the separation of variables and employing the convolution integral theorem in conjunction with the impulse response function, for a general forcing function $P(t)$ (Ref. 5):

$$w_i(x,t) = w_i(x)\int_0^t P_i(\tau)e^{-\xi_i\omega_i(t-\tau)}\sin\omega_{id}(t-\tau)\,\mathrm{d}\tau \tag{9}$$

where ω_i is undamped natural frequency, ω_{id} is the damped natural frequency, ξ_i is the equivalent viscous damping, and $P_i(\tau)$ is the excitation force in the ith natural mode, i.e., ϕ_i, which is defined as

$$P_i(\tau) = \int_0^L P(\bar{x},\tau)\phi_i(\bar{x})\,\mathrm{d}\bar{x} \tag{10}$$

For a lightly damped structure, the total structural response $w(x,t)$ can be obtained by the mode superposition principle[5] inasmuch as the modal phase differences are small and can be neglected. The dynamic load $P(\tau)$ is taken to be a point load, applied impulsively to represent either an angle-of-attack disturbance or a control force application situation. Further, as both these forces generally act close to the tip and the root of the space vehicle, respectively, these two locations are selected for applying the impulse excitation.

Finally, elastic displacement is important not only for the structural design but also for the control sensors, and, therefore, the peak values of the displacement w are obtained as functions of two joint parameters $\bar{F}_j$ ($=F_j EI_0/L_0$, flexibility) and $\bar{x}_j$ ($=x_j/L_0$, location). It can be shown that, whereas the peak values in time occur very close to $t=0$, their spanwise locations depend on the resultant superposed deformed displacement shape. Further, the magnitude of transient peaks are found to be independent of ξ for small values, i.e., <0.10, and, therefore, in the present study all of the modes are assumed to possess an equivalent viscous damping ratio as 1% of the critical damping factor. Last, a convergence study has shown that inclusion of the first six vibration modes in the response calculation is adequate.

Results and Discussion

Figures 2a and 2b present the normalized results for the first two natural frequencies of the generic space vehicle as functions of $\bar{F}_j$ (values from 0.0 to 0.2) and $\bar{x}_j$ (values of 0.348, 0.432, 0.511, 0.723, 0.850, and 0.935). It can be seen that, whereas the maximum reduction in λ for the first mode is around 23%, the same for the second mode is only around 11%. It has been found that, for higher modes, the maximum effect of a joint flexibility of 0.15 on λ is even lower. However, in both of these cases, $\bar{\lambda}_i$ can be reasonably approximated by a straight-line function of $\bar{F}_j$, particularly for lower values of $\bar{F}_j$. In the present case, analytical expressions have been obtained for

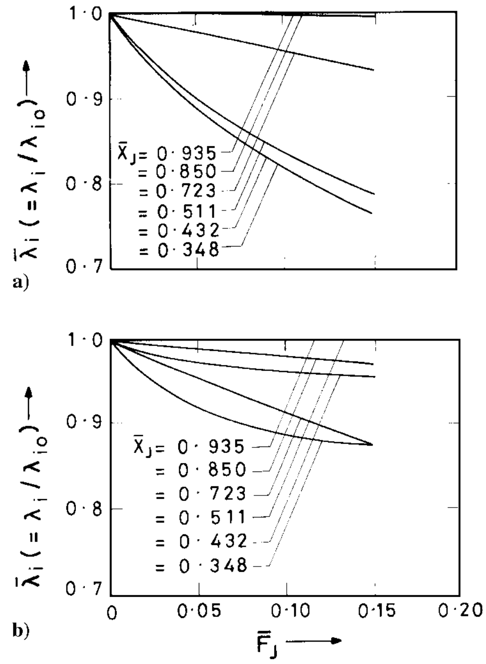

Fig. 2 Variation of normalized frequency parameter $\bar{\lambda}$ vs $\bar{F}_j$ for different values of location parameter $\bar{x}_j$, for a) mode 1 and b) mode 2.

the first vibration mode frequency parameter, by fitting a straight line among the data points close to the low values of $\bar{F}_j$, which are

$$\begin{aligned}
\bar{\lambda}_1 &= 1.0 - 2.32\bar{F}_j \quad \text{for} \quad \bar{x}_j = 0.348\\
\bar{\lambda}_1 &= 1.0 - 2.38\bar{F}_j \quad \text{for} \quad \bar{x}_j = 0.432\\
\bar{\lambda}_1 &= 1.0 - 2.06\bar{F}_j \quad \text{for} \quad \bar{x}_j = 0.511\\
\bar{\lambda}_1 &= 1.0 - 0.48\bar{F}_j \quad \text{for} \quad \bar{x}_j = 0.723\\
\bar{\lambda}_1 &= 1.0 - 0.04\bar{F}_j \quad \text{for} \quad \bar{x}_j = 0.850\\
\bar{\lambda}_1 &= 1.0 \quad \text{for} \quad \bar{x}_j = 0.935
\end{aligned} \tag{11}$$

It is seen from these six relations that, as the degraded joint moves from the root ($\bar{x}_j = 0.0$) to the tip ($\bar{x}_j = 1.0$), its influence on the first mode frequency parameter decreases nearly monotonically.

Thus, it is possible to relate this trend to the $EI(x)$ distribution in the generic space vehicle that, in general, also reduces nearly monotonically from the root to the tip. A simple functional form to unify Eq. (11) and to relate the six constants in Eq. (11) to the basic structural stiffness distribution of the space vehicle can be given as

$$\bar{\lambda}_1 = 1.0 - \alpha[EI(\bar{x}_j)]^{\frac{1}{4}}\bar{F}_j, \qquad \alpha = 1.42 \quad \text{for} \quad \bar{x}_j < 0.5$$

$$\alpha = 0.03 \quad \text{for} \quad \bar{x}_j > 0.7 \tag{12}$$

It is seen from Fig. 2 that there is no well-defined trend with respect to $\bar{x}_j$ for a given $\bar{F}_j$, which may be because the influence of $\bar{x}_j$ depends on the contribution of each mode to the total response. Figures 3a and 3b show the transient elastic response in terms of normalized peak values of displacement (w_{norm}), due to a 1.0-g impulse excitation at both root and tip stations as a function of $\bar{F}_j$ for various values of $\bar{x}_j$. These results show that there also is no well-defined trend for the peak response, except for a small region around the origin, and this also can be attributed to the varying degree of modal contribution to the total response. Finally, it is interesting to note from Fig. 3 that excitation at the tip ($\bar{x} = 1.0$) produces a substantially larger response than the same excitation at the root ($\bar{x} = 0.0$) for the same flexibility and location of a degraded joint. This can be attributed to generation of larger moments near the mode shape peak for the tip excitation, and this makes an angle-of-attack

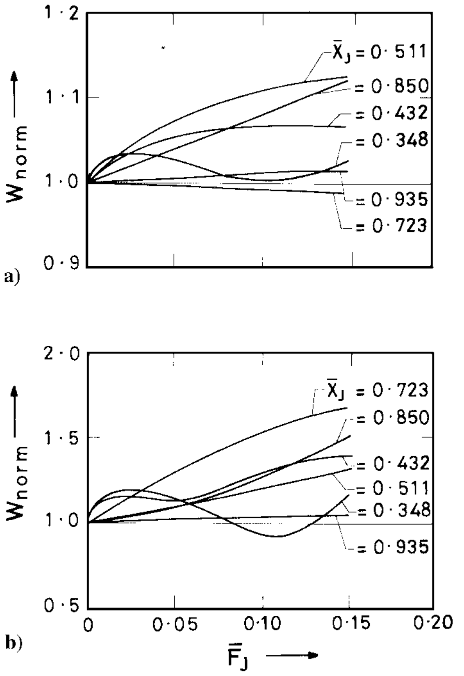

Fig. 3 Variation of w_{norm} with joint flexibility parameter $\bar{F}_j$ for a) root impulse and b) tip impulse.

disturbance structurally more critical than a control impulse of the same magnitude.

Conclusions

The present study has investigated the problem of the elastic response of a generic space vehicle with degraded joints. The stepped-beam formulation based on elementary beam theory is employed for extracting modal parameters of a normalized generic space vehicle. Next, the modal superposition principle is used with a low damping value for obtaining the elastic response of the vehicle to impulse excitation.

The results for elastic displacement peaks are obtained for normalized rotational joint flexibilities of different magnitudes acting at different spanwise locations. The results show that locations of the joint with flexibility play an important role in determining the magnitude and location of the peak response. Further, it is found that first mode frequency results for various values and locations of joint flexibilities can possibly be unified into a single linear expression, based on the normalized stiffness distribution of the space vehicle. Finally, it is found that tip excitation is more critical than root excitation.

Acknowledgment

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Aerodynamics of a Spinning Cylinder in Rarefied Gas Flows

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Introduction

INCOMPRESSIBLE flows around spinning bodies of revolution were studied in detail years ago (see the reviews by Prandtl and Tietjens[1] and Lugt[2]). It was found that, in the case of potential flow, the lift generated on the body has an opposite direction to the vector $[\boldsymbol{\Omega} \times \boldsymbol{u}_\infty]$ (the Magnus effect), where $\boldsymbol{\Omega}$ is an angular rotation vector of the spinning body and $\boldsymbol{u}_\infty$ is a freestream flow velocity vector. In this flow regime, three types of the flow patterns past a spinning circular cylinder can be identified[1,2] by the value of the governing similarity parameter, which is the roll parameter $\Theta = \Omega D/2u_\infty$, where D is a diameter of a cylinder. The patterns depend on the location of the points of separation and attachment.

In unsteady flow of a viscous incompressible fluid, the flow pattern becomes a function of both the roll parameter and the Reynolds number.[2] At $Re < 1.3 \times 10^5$ and $\Theta < 0.5$, the Magnus force can become negative.[3]

In the case of free-molecule flow, a different result was observed in Refs. 4–6. The lift of the spinning body under the free-molecule flow conditions should be opposite to the vector of lift under the continuum incompressible (potential) flow conditions. The lift and drag coefficients C_y and C_x, respectively, can be calculated using the formulas[6]

$$
\begin{aligned}
C_y(\Theta) &= (\pi/2)\sigma_t\Theta, \qquad & C_x(\Theta) &= 0 \\
C_y &= C_y(0) + C_y(\Theta), \qquad & C_x &= C_x(0) + C_x(\Theta)
\end{aligned}
\tag{1}
$$

where the parameter σ_t is the coefficient of accommodation of the tangential momentum. The lift and drag coefficients $C_y(0)$ and $C_x(0)$ in the nonspinning-cylinder case have been calculated by using the following expressions[7]:

$$
\begin{aligned}
&C_y(0) = 0, \qquad C_x(0) = C_{x,i}(0) + C_{x,r}(0) \\
&C_{x,i}(0) = \frac{\sqrt{\pi}}{S}e^{-(S^2/2)}\left\{I_0\left(\frac{S^2}{2}\right) + \frac{1+2S^2}{2}\left[I_0\left(\frac{S^2}{2}\right) + I_1\left(\frac{S^2}{2}\right)\right]\right\} \\
&C_{x,r}(0) = \frac{\pi^{3/2}}{4u_\infty\sqrt{h_r}}, \qquad S = \sqrt{\frac{\gamma}{2}}M_\infty, \qquad h_r = \frac{m}{2kT_r}
\end{aligned}
\tag{2}
$$

where M_∞ is the Mach number, S is the molecular speed ratio, γ is a ratio of specific heats, m is a mass of molecule, k is Boltzmann's constant, T is temperature, and I_0 and I_1 are modified Bessel functions. Subscripts i and r refer to incident and reflected molecules, respectively. The Cartesian coordinate x is in the direction of the freestream flow velocity vector, and the coordinate y is in the direction of the vector $[\boldsymbol{\Omega} \times \boldsymbol{u}_\infty]$.

Karr and Yen[4] showed that the effect of spin on drag is of second order in Θ, and the component of lift $C_y(\Theta)$ has been found to be proportional to Θ and is analogous to the Magnus effect with the opposite sign.[6] The expressions of the momentum characteristics can be found in Ref. 6.

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